

is nearly constant. The decay of peak tangential velocities given in Eq. (4) is substantiated by empirical results.

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A Closed-Form Solution to Oblique Shock-Wave Properties

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THIS Note is concerned with the direct computation of oblique shock-wave properties with freestream Mach number and flow-deflection angle as the independent variables.

The equations governing oblique shock relations may be written in the form

$$\sin^6\theta + b \sin^4\theta + c \sin^2\theta + d = 0$$

where

$$b = - \left[\frac{M_1^2 + 2}{M_1^2} \right] - \gamma \sin^2\delta$$

$$c = (2M_1^2 + 1)/M_1^4 + [(\gamma + 1)^2/4 + (\gamma - 1)/M_1^2] \sin^2\delta$$

$$d = - \cos^2\delta/M_1^4$$

and

- θ = shock-wave angle
- M_1 = freestream Mach number
- δ = deflection angle
- γ = ratio of specific heats

which is cubic in $\sin^2\theta$, having three real roots, the smallest of which results in a decrease in entropy.

Contrary to the statement of Ref. 1, that no convenient explicit relation exists for this case, there is indeed a general solution for a cubic. The mathematical derivation can be found in Ref. 2. From Ref. 2, the solution for a cubic having three real roots is

$$\sin^2\theta = -b/3 + \frac{2}{3} (b^2 - 3c)^{1/2} \cos[(\phi + n\pi)/3]$$

where

$$\cos\phi = (\frac{3}{2}bc - b^3 - \frac{27}{2}d)/(b^2 - 3c)^{3/2}$$

and $n = 0$ corresponds to the strong shock solution; $n = 2$ results in a decrease in entropy; and $n = 4$ corresponds to the weak shock solution. Although many readers may be aware of this solution, the wide use of iteration schemes to solve this problem has prompted the author to set down the explicit solution in general terms.

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Free Vibration of Simply Supported Parallelogrammic Plates

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Nomenclature

- a, b = dimensions of the plate, see Fig. 1a
- D = plate rigidity, $Eh^3/12(1 - \nu^2)$
- E = Young's modulus of the material of the plate
- h = plate thickness
- k = frequency parameter, $(\rho h/D)^{1/2} \omega a^2/\pi^2$
- k_m = frequency parameter of membrane, $(\mu/S)^{1/2} \omega a/\pi$
- m, n = number of half sine waves in the two directions x_1 and y_1 , respectively
- S = uniform tension per unit length of stretched membrane
- x, y = rectangular coordinate system defined in Fig. 1a
- x_1, y_1 = oblique coordinates defined in Fig. 1a
- ρ = mass density of the plate material
- ψ = angle of skew, defined in Fig. 1a
- ω = frequency of oscillation in rad/sec
- μ = mass per unit area of membrane
- ν = Poisson's ratio

Introduction

IN this Note, the results of numerical calculations for the first few frequencies of simply supported parallelogrammic plates, using the Rayleigh-Ritz method employing double Fourier sine series in oblique coordinates, are presented. Interesting features, hitherto unreported in the literature, such as 1) the skew angle splitting the degenerate frequencies of rectangular plates to distinct ones and 2) the "frequency crossing" of the modes of simply supported skew plates, are discussed. In fact, it has been shown in Ref. 1 that these features are also exhibited by clamped skew plates.

The literature does not contain adequate results for the frequencies of simply supported parallelogrammic plates. Conway and Farnham² calculated only the fundamental frequency for a few configurations of the plate by point matching, using a mathematical relationship that exists between the problems of a simply supported polygonal plate and a polygonal membrane of the same geometry.³⁻⁵ This relationship shows that the eigenvalues of the plate are squares of the eigenvalues of the membrane, whereas the eigenfunctions are identical. Weinstein⁶ reports the upper and lower bounds of the frequencies of modes symmetric about both the diagonals of rhombic membrane, which have been calculated by Stadter⁷ in an unpublished report. These values serve admirably for comparison with the results of simply supported rhombic plates on the basis of the aforementioned relationship.

Details of Solution

The vibration problem becomes a particular case of panel-flutter problem of simply supported parallelogrammic panels, which is discussed in detail in Ref. 8. Consequently, the

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